MHF 4U Final Review Sheets

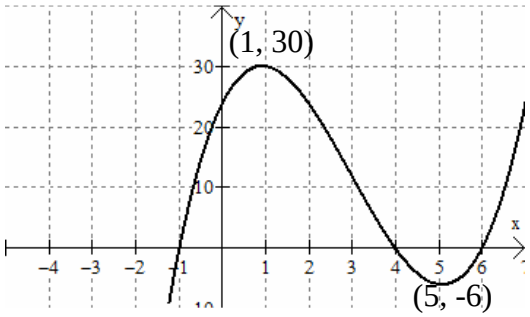
1. Sketch each polynomial function:

a) $y=\frac{1}{2}(x-2)^3-3$ b) $y=-2x^2(x+3)$ c) $y=(x+1)(x-2)^3$

d) $y=x^4-13x^2+36$

2. Given $y=f(x)$

a) State the degree of the function.
b) State the coordinates of the zeros, local minimums and local maximums.
c) State the intervals of increasing and decreasing.
d) Find the equation of the function



3. a) Find the quotient and the remainder when $3x^3-4x+3$ is divided by $x-2$.
b) Find the value of k in x^3-2x^2+kx-3 if $(x+3)$ is a factor.

4. Factor a) $x^3-4x^2-4x+16$ b) $8x^3+27$ c) $5x^3+3x^2-12x+4$ d) x^4-11x^2+18

5. Solve each: a) $2x^3+x^2-18x-9=0$ b) $x(x^2+9x+3)=-5(2x+1)$
c) $-2x(x+3)(x-2)(x-5)<0$ d) $-x^3-5x^2+6x\geq 0$

6. Find the cubic polynomial function with two of its zeros 2 and $-3+\sqrt{2}$, y-intercept of 7.

7. a) Find the value of k if $2x^3-4x^2-3x+k$ is divisible by $2x-3$.
b) Given ax^3+x^2+x+b, find the value of a and b if the remainder when divided by $(x-1)$ and $(x+1)$ are 6 and 2 respectively.

8. Given $y=x^3-2x+3$

a) Find the average rate of change of y with respect to x in the interval [-2, 3]
b) Estimate the slope of the tangent to the curve at $x=3$
c) Find the equation of the tangent at at x =2.

9. State the asymptotes to each curve:

a) $y=\frac{3x}{x^2-4}$ b) $y=\frac{2x^3+x^2+5}{x^2}$ c) $y=\frac{2x^2-18}{x^2+x-12}$ d) $y=\frac{x^2+5x-1}{x+2}$

e) $y=2^{x+1}-3$

10. Analyze each rational function the sketch the graph. Consider the domain, x and y-intercepts, asymptotes and symmetry.

a) $y=\frac{2}{x+3}-2$ b) $y=\frac{2x+6}{x-3}$ c) $y=\frac{x}{x^2-4}$

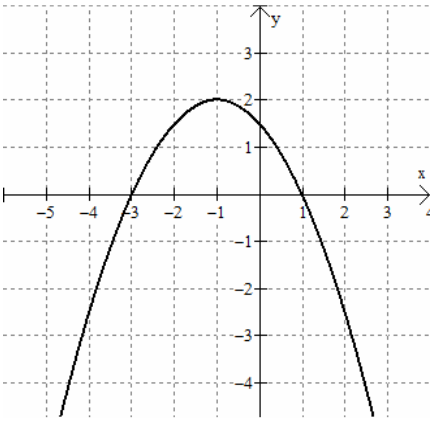
11. Solve the following rational equations and inequalities.

a) $\dfrac{2x^2+5x-12}{x+1}=0$ b) $\dfrac{2x}{6x+5}=\dfrac{x+3}{3x-1}$ c) $\dfrac{3x+2}{2x+1}=\dfrac{3x+1}{x-1}-\dfrac{1}{3}$

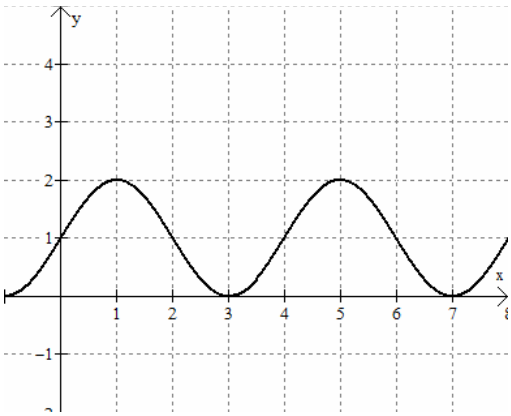
d) $\dfrac{-x+5}{2x+3}\geq 2$ e) $\dfrac{1}{x-1}<\dfrac{-1}{x+2}$ f) $\dfrac{x}{x-2}\leq\dfrac{x-1}{x+1}$

12. Given $y=f(x)$ sketch the graph of the reciprocal function $y=\dfrac{1}{f(x)}$

a



b



13. The function $P(t)=\dfrac{25(5t+8)}{2t+1}$ models the population, in thousands, of a town *t* years since 1990. Find: a) the initial population.

b) the average rate of change of the population from 1995 to 2005.

c) the rate at which the population is changing in the year 2000.

14. a) Convert to radians i) 75° ii) 330° b) Convert to degrees: i) $\frac{3\pi}{5}$ ii) 4.5^{r}

15. Find the exact value of: a) $\sin\dfrac{4\pi}{3}$ b) $\cot\dfrac{5\pi}{6}$ c) $\sec\dfrac{7\pi}{4}$

16. Solve for **all** θ, $0\leq x\leq 2\pi$. Find exact values where possible.

a) $\cot\theta=\sqrt{3}$ b) $\sqrt{2}\cos\theta+1=0$ c) $\sin\theta=0.325$

d) $\sin(2\theta)=\dfrac{\sqrt{3}}{2}$

e) $3\sin^2\theta-5\sin\theta-2=0$ f) $2\cos^2\theta-\cos\theta=0$ g) $2\sin^2\theta=\cos\theta$

h) $\sin(2\theta)-\cos\theta=0$ i) $2\cos\frac{\pi}{3}(\theta-1)+5=4$

17. Simplify $\cos\left(\dfrac{3\pi}{4}+x\right)+\sin\left(\dfrac{3\pi}{4}-x\right)$

18. Find the exact value of : a) $\sin\dfrac{\pi}{12}$ b) $\cos\dfrac{13\pi}{12}$.

19. If $\cos\theta=\dfrac{-2}{\sqrt{7}}$, $0\leq\theta\leq\pi$ find the exact value of $\cos 2\theta$ and $\sin 2\theta$.

20. Prove: a) $\tan\theta+1=\dfrac{\cos^2\theta-\sin^2\theta}{\cos^2\theta-\sin\theta\cos\theta}$ b) $\sec^2\theta+\sin^2\theta-2=\tan^2\theta-\cos^2\theta$

c) $(\sec x-\cos x)(\csc x-\sin x)=\dfrac{\tan x}{1+\tan^2 x}$ d) $\sin 2A=\dfrac{2\tan A}{1+\tan^2 A}$

e) $\cot\theta+\tan\theta=2\csc 2\theta$ f) $\dfrac{\sin x+\sin 2x}{1+\cos x+\cos 2x}=\tan x$

g) $\sin(x+y)\sin(x-y)=\cos^2 y-\cos^2 x$

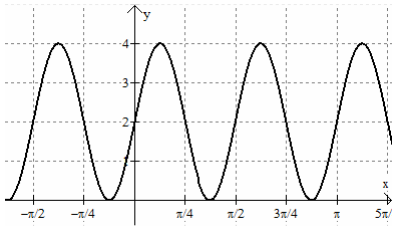
21. For each, state the amplitude, period, phase shift and sketch the graph.

a) $y=2\sin\dfrac{1}{3}\left(x-\dfrac{\pi}{3}\right)$ b) $y=-\cos\left(2x+\dfrac{\pi}{2}\right)+1$ c) $y=2\cos\dfrac{\pi}{6}x+5$

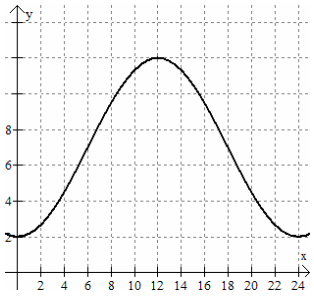
22. State the equation of each sine function.

a) amplitude of 3.5, period of $\dfrac{2\pi}{5}$, phase shift of $\dfrac{\pi}{4}$

b)



c)



23. The points $\left(\dfrac{\pi}{2},0\right)$ and $(\pi,-1)$ are on the curve $y=\cos\theta$. State the new coordinates of these points under the transformations given by $y=3\cos\dfrac{2}{5}\left(\theta-\dfrac{\pi}{4}\right)-1$.

24. The average monthly temperature, T, in degrees Celsius, for any month, t, for the town of Someplace, is modelled by the function $T(t)=19.1\sin\left(\dfrac{\pi t}{6}+4.2\right)+7.5$. For $t=0$, the month is January.

a) What is the maximum average monthly temperature?
b) What is the period of the function and what does it mean?
c) Determine the month with minimum average monthly temperature.
d) When is the average monthly temperature about $24°C$.

25. A pedal on a bicycle has an arm length of 20 cm and rotates about an axle 32 cm above the ground. If the pedal states at its lowest point and rotates at 20 revolutions every minute, find a sinusoidal function that will model the height *h*, in centimetres, of the pedal after *t* seconds. At what time during the first 5 seconds will the pedal be 40 cm above the ground?

$y=-20\cos\left(\frac{2\pi}{3}x\right)+32$, $t=0.946s, 2.043s, 3.924s$

26. Evaluate without a calculator:

a) $\log_3 \frac{1}{27}$ b) $\log_5 \sqrt[3]{625}$ c) $\log_8 32$ d) $\log 0.001$ e) $\log 25 - \log \frac{5}{2}$

27. Simplify: a) $\log_a \sqrt{a^5}$ b) $\log_a \frac{1}{\sqrt[5]{a^2}}$ c) $4^{3\log_4 x}$ d) $3\log 2x - 5\log x + 2\log 2$

e) $(\log 10^x)(\log a + 2\log b)$

28. For each function:

i) State the domain and range ii) sketch the graph iii) state the equation of its inverse

a) $y = -3^{x-2} - 1$ b) $y = -2\log_3(x+1)$ c) $y = 1 + \log_2(x-4)$

29. Solve for x: a) $7^x = 21$ b) $4(3^{2x}) - 5 = 29$ c) $3^{x-1} = 5^{1-2x}$ d) $5\log_3 x - 8 = 22$

e) $2\log_x 3 = 3$ f) $\log(x^2+4) = 1 + 2\log x$ g) $\log_2(3x+2) = 5 - \log_2(x-9)$

h) $5\log(x-3) = 2$ i) $\log_2(\log x) = 3$

30. a) A radioactive substance has a half-life of 250 years. Determine the age of the substance if it has retained 74% of its original amount.

b) If a population increases at a rate of 1.5% per year, find its doubling period.

c) 96.2 g of radioactive material is placed in a nuclear reactor. After 8 days only 35% remains. Determine the half-life of the substance.

d) A car depreciates at 15% per year. After how many years will it be worth half of its original value?

e) A bacterial culture starts with 1000 bacteria and triples after 5 hours. Find the doubling period and the amount of bacteria after 8 hours.

31. The half-life of a radioactive substance is 56 years. If there is 250 mg of the substance now:

a) Find the average rate of decay per year over the first 30 years

b) Estimate the rate of decay of the substance after 100 years.

32. Given $f(x) = 2x^2 - x$, $g(x) = \frac{x}{2-x}$, $h(x) = x + \cos x$ and $k(x) = \log(x-1)$,

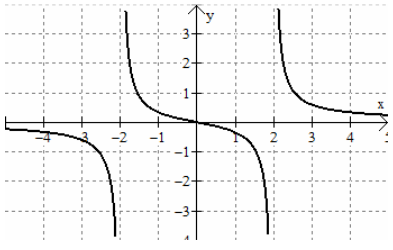
i) find each function and state its domain:

a) $f(g(x))$ b) $h \circ g(x)$ c) $g \circ k(x)$ d) $(k \circ f)(x)$ e) $(fg)(x)$ f) $\left(\frac{g}{f}\right)(x)$

ii) find x if $g(f(x)) = 1$

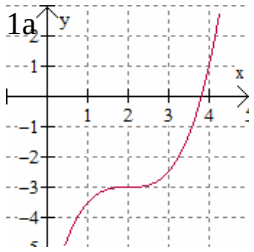
33. Determine if the given functions are even, odd or neither. Support your answer.

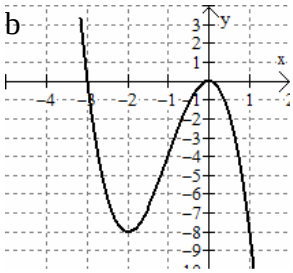
a) $y = x^3 + 4x$ b) $y = 3x^4 + 5x^2 - 1$ c) $y = \frac{x}{x-4}$ d) $y = 3\sin x$ e) $y = \cos 2x$

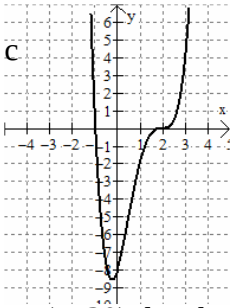
f) 

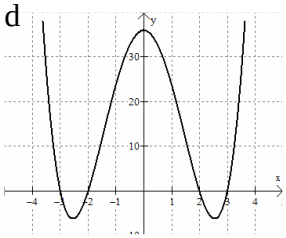
g) $y = 2^{x+4}$ h) $y = \frac{\sin x}{x}$ i) $y = x^2 + \sin x$

Answers:

1a 

b 

c 

d 

2. a) 3rd degree b) zeros (-1,0), (4,0) and (6,0) local max (1, 30), local minimum (5, -6) c) increasing $x<1$ or $x>5$, decreasing $1<x<5$ d) $y=k(x+1)(x-4)(x-6), k=1$

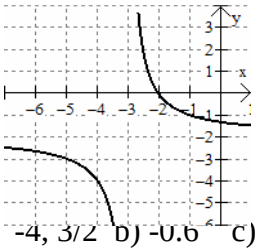
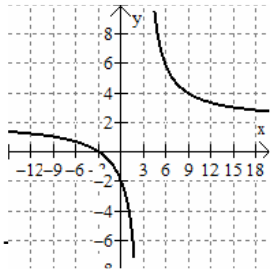
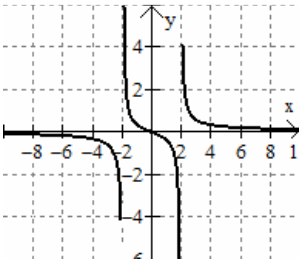
3.a) $3x^2+6x+8,\ R:19$ b) $k=-16$

4. a) $(x-2)(x+2)(x-4)$ b) $(2x+3)(4x^2-6x+9)$ c) $(x+2)(x-1)(5x-2)$ d) $(x-3)(x+3)(x^2-2)$ **5.** a) $\pm 3, -\frac{1}{2}$ b) $-1,\ -4\pm\sqrt{11}$ c) $x<-3,\ 0<x<2,\ x>5$ d) $x\le -6,\ 0\le x\le 1$

6. $y=-0.5(x-2)(x^2+6x+7)$ **7.** a) $\frac{27}{4}$ b) $a=1,\ b=3$

8. a) $\frac{\Delta y}{\Delta x}=5$ b) $\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=25$ c) $y=10x-13$

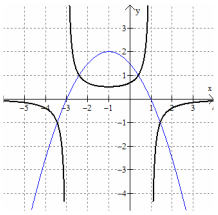
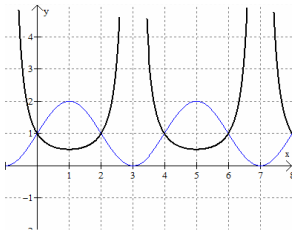
9. a) $x=\pm 2, y=0$ b) $x=0, y=2x+1$ c) $x=-4, y=2$ d) $x=-2, y=x+3$ e) $y=-3$

10.a)  b)  c) 

11. a) -4, 3/2 b) -0.6 c) -5/7, -

d) $-\frac{3}{2}\le x\le -\frac{1}{5}$ e) $x<-2$ or $-\frac{1}{2}<x<1$

f) $x<-1$ or $\frac{1}{2}\le x<2$

12. a  b 

13. a) 200 000 people b) decreasing 806 people/year c) decreasing at 624 people/year

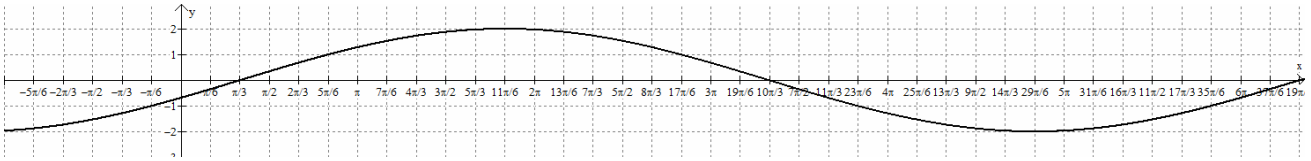
14. a) i) $\frac{5\pi}{12}$ ii) $\frac{11\pi}{6}$ b) i) 108° ii) 258° **15.** a) $-\frac{\sqrt{3}}{2}$ b) $-\sqrt{3}$ c) $\sqrt{2}$

16. a) $\frac{\pi}{6}, \frac{7\pi}{6}$ b) $\frac{3\pi}{4}, \frac{5\pi}{4}$ c) 0.331, 2.811 d) $\frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}, \frac{4\pi}{3}$ e) 3.48, 5.94 f) $\frac{\pi}{3}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{3}$

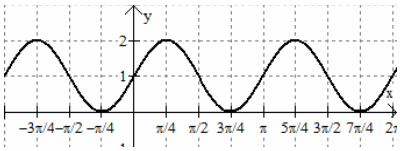
g) 0.675, 5.60 h) $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$ i) 3, 5

17. 0 **18.** a) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ b) $\frac{-1-\sqrt{3}}{2\sqrt{2}}$ **19.** $\frac{1}{7}$, $\frac{-4\sqrt{3}}{7}$

21 a) amp = 2, per = 6π p.s. = right $\frac{\pi}{3}$

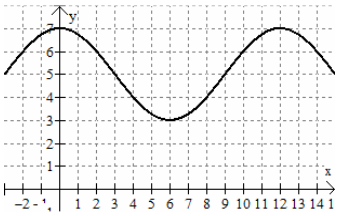


21 b) amp = 1, per = π p.s. = left $\frac{\pi}{4}$



c)

amp = 2
Per = 12
p.s. none



22. a) $y = 3.5\sin 5(x-\frac{\pi}{4})$ b) $y = 2\sin(4x)+2$ c) $y = 5\sin\frac{\pi}{12}(x-6)+7$

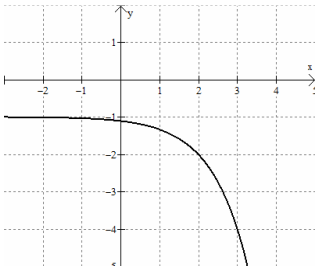
23. $\left(\frac{3\pi}{2},-1\right)$, $\left(\frac{11\pi}{4}-4\right)$ **24.** a) 26.6° b) 12, annual cycle c) $t = 0.9786 \therefore$ Feb d) June, Aug

25. $y = -20\cos\left(\frac{2\pi}{3}t\right)+32$, $t = 0.946s, 2.053s, 3.924s$

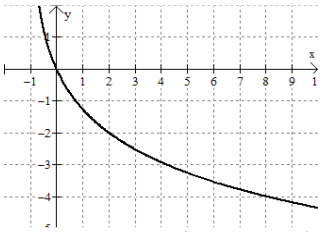
26. a) -3 b) $\frac{4}{3}$ c) $\frac{5}{3}$ d) -3 e) 1

27. a) $\frac{5}{2}$ b) $-\frac{2}{5}$ c) x^3 d) $\log\frac{32}{x^2}$ e) $\log a^x b^{2x}$

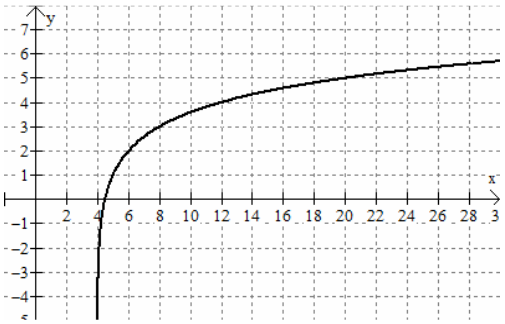
28 a) $x \in R, y < -1, y \in R$, $f^{-1}(x) = \log_3(-x-1)+2$



b) $x > -1, x \in R, y \in R$, $f^{-1}(x) = 3^{-\frac{x}{2}}-1$



28 c) $x > 4, x \in R, y \in R$, $f^{-1}(x) = 2^{x-1}+4$



29. a) 1.565 b) 0.974 c) 0.627 d) 729

e) 2.08 f) $\frac{2}{3}$ g) 10 h) 5.512 i) 10^8

30 a) 108.6 yrs b) 46.6 yrs c) 5.28 d

d) 4.27 yrs e) 3.15 hrs, 5800 bacteria

31. a) 2.58 mg/year b) 0.9 mg/year

32. i) a) $\frac{3x^2-2x}{(2-x)^2}$, $x \neq 2$ b) $\frac{x}{2-x}+\cos\left(\frac{x}{2-x}\right)$, $x \neq 2$ c) $\frac{\log(x-1)}{2-\log(x-1)}$, $x>1$

d) $\log(2x^2-x-1)$ $x<-0.5$ or $x>1$ e) $\frac{x^2(2x-1)}{2-x}, x \neq 2$ f) $\frac{1}{(2x-1)(2-x)}, x \neq 0, \frac{1}{2}, 2$

ii) $-\frac{1}{2}$, 1

33. a) odd b) even c) neither d) odd e) even f) odd g) neither h) even i) neither